A real time static potential for quarkonium properties at finite T

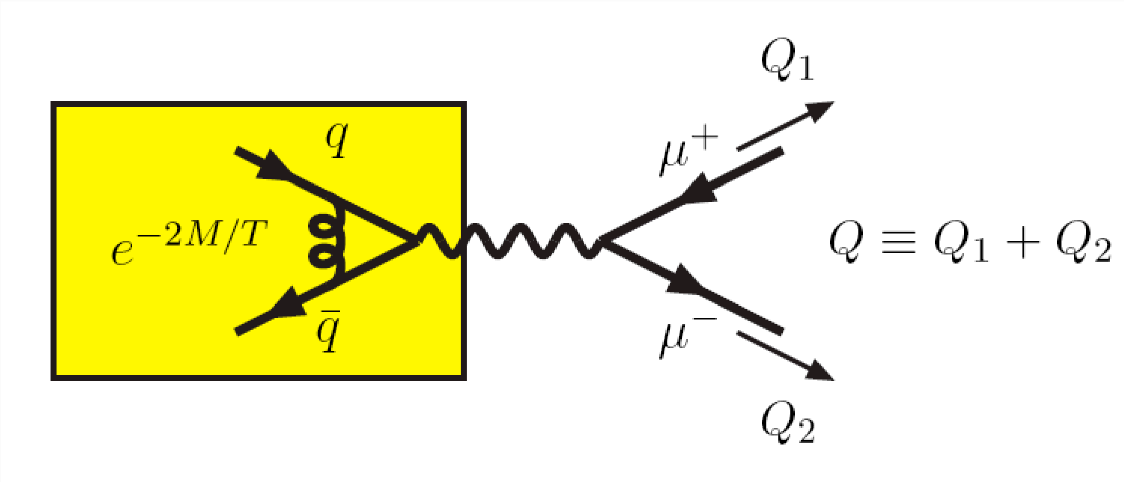
Owe Philipsen

WESTFÄLISCHE WILHELMS-UNIVERSITÄT MÜNSTER

in collaborations with O. Jahn, M. Laine, P. Romatschke, M. Tassler

- Critical assessment of lattice potentials used in potential models
- A new real time potential
- Perturbative + non-perturbative aspects

Quarkonia in the QCD plasma



$$\frac{\mathrm{d}N_{\mu^+\mu^-}}{\mathrm{d}^4x\,\mathrm{d}^4Q} \propto \frac{\rho_V(\omega)}{\omega^2} e^{-\frac{\omega}{T}}$$

Lattice: engineer spectral fcn. from euclidean correlators via MEM

“MEM is not reliable for reconstructing spectral functions at finite T”

Peter Petreczky, Non-Equilibrium Dynamics, KITP 08, slide 8

➡ also pursue other approaches, if only to improve default models for MEM

Here: potential models ⟷ QFT ?

Potential models

T=0: solve static Schrödinger eqn. with confining V (from the lattice)

$$\left(-\frac{\nabla_{\mathbf{r}}^2}{M}+V(r)\right)\psi=(E-2M)\psi$$

very successful spectroscopy (~1%), search for hybrids, ...

can be derived from QFT in effective theory → pNRQCD
expansion parameter $(E-2M)/M$;

V = perturbative matching coefficient

finite T: argue for a 'finite T potential', proceed as above $V(r)\approx-\frac{g^2C_F}{4\pi}\frac{\exp(-m_{\rm D}r)}{r}$

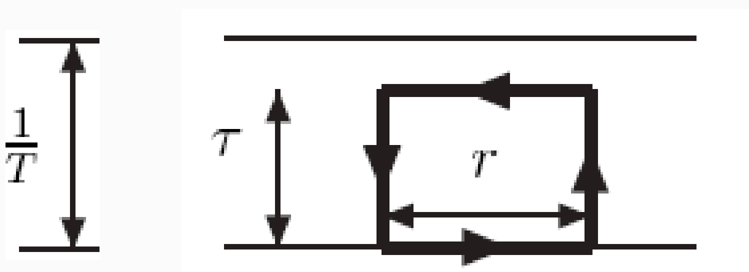
Problems:

-matching coeff. not the same as non-pert. static potential
-which non-pert. potential to take? → talk A. Mocsy

The static potential at T=0: Wilson loop

- Euclidean correlator of gauge invariant meson operator
- integrate out quarks in the limit $M \to \infty$

$$\langle \bar{\psi}(\mathbf{x},\tau) U(\mathbf{x},\mathbf{y};\tau) \psi(\mathbf{y},\tau) \bar{\psi}(\mathbf{y},0) U^\dagger(\mathbf{x},\mathbf{y};0) \psi(\mathbf{x},0) \rangle \longrightarrow \mathrm{e}^{-2M\tau} W_E(|\mathbf{x}-\mathbf{y}|,\tau)$$



$U(\mathbf{x},\mathbf{y};0) \equiv U_r(0)$

$$r = |\mathbf{x}-\mathbf{y}|$$

- spectral decomposition, $T \to 0$ $\qquad W_E(r,\tau \to \infty) \longrightarrow c_{01}^2 [U(\mathbf{x},\mathbf{y};0)]\, \mathrm{e}^{-V(r)\tau}$
- lattice generalization to finite T not clear, $N_\tau = \dfrac{1}{aT}$ on lattices short

Static 'potentials' at finite T, free energies: the Polyakov loop

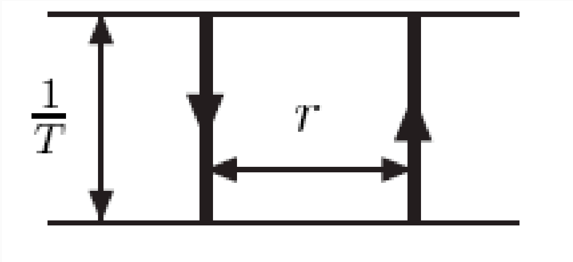
McLerran, Svetitsky PRD 81

- Static quarks propagate through periodic boundary
- finite T: sum over Boltzmann weighted excited states

➡ free energy of a static quark in a plasma: $\langle \mathrm{Tr}\, L_{\mathbf{x}} \rangle \sim \mathrm{e}^{-F_q/T}$

order parameter for confinement: $\langle L \rangle \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, & T > T_c \end{cases}$

- free energy of static quark anti-quark pair:



$$\mathrm{e}^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{N^2} \langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y}) \rangle, \quad r = |\mathbf{x} - \mathbf{y}|$$

Decomposition in different colour channels

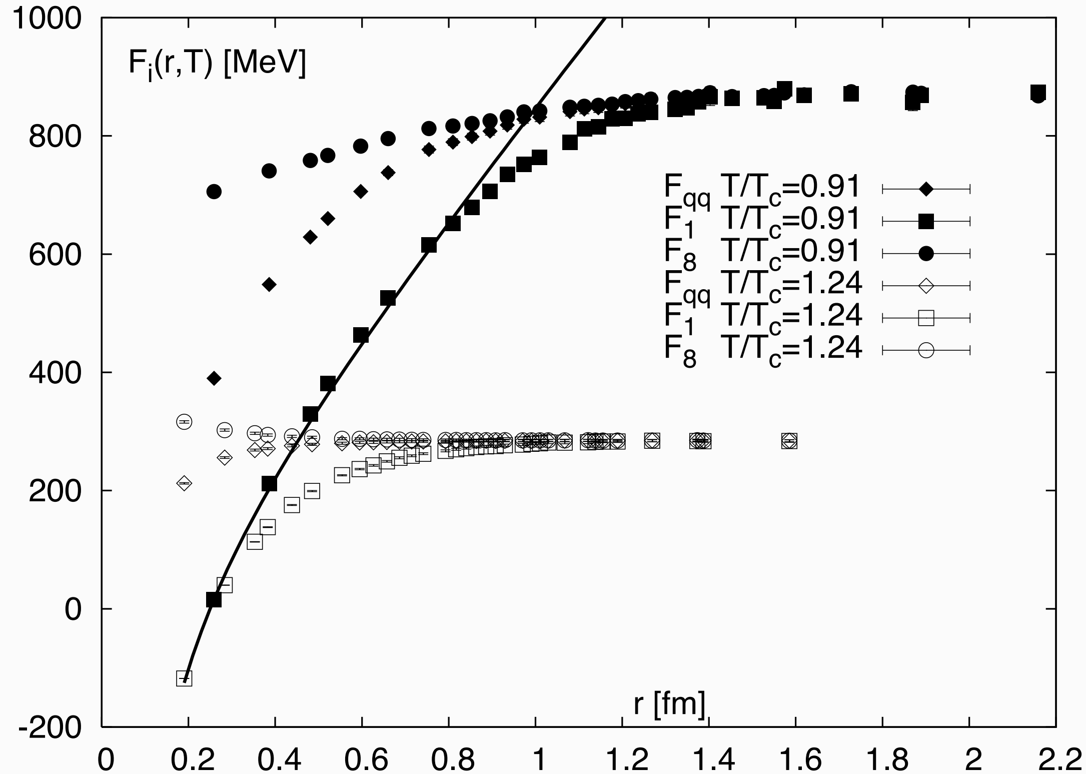
McLerran, Svetitsky PRD 81

$$
\begin{aligned}
\mathrm{e}^{-F_{\bar{q}q}(r,T)/T} &= \frac{1}{N^2}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\,\mathrm{Tr}\, L(\mathbf{y})\rangle = \frac{1}{N^2}\mathrm{e}^{-F_1(r,T)/T} + \frac{N^2-1}{N^2}\mathrm{e}^{-F_8(r,T)/T} \\
\mathrm{e}^{-F_1(r,T)/T} &= \frac{1}{N}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})L(\mathbf{y})\rangle, \\
\mathrm{e}^{-F_8(r,T)/T} &= \frac{1}{N^2-1}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\,\mathrm{Tr}\, L(\mathbf{y})\rangle - \frac{1}{N(N^2-1)}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})L(\mathbf{y})\rangle.
\end{aligned}
$$

- correlators in ‘singlet’ and ‘octet’ channels gauge dependent
- $$F_1(r,T) \sim \frac{\mathrm{e}^{-m_D(T)r}}{4\pi r}$$ Nadkarni PRD 86
- non-perturbative meaning?

Numerical results in Coulomb gauge

Kaczmarek, Zantow PRD 05



- \`Potentiology': solve Schrödinger with various potentials, check if solution allows to reconstruct lattice correlators

$$F_i, \quad U_i = F_i + TS_i \ldots$$

- different r-dependence results in different binding behaviour in Schrödinger eq.

- F_1, U_1 gauge artefact or physics?

Spectral analysis of Polyakov loop correlators

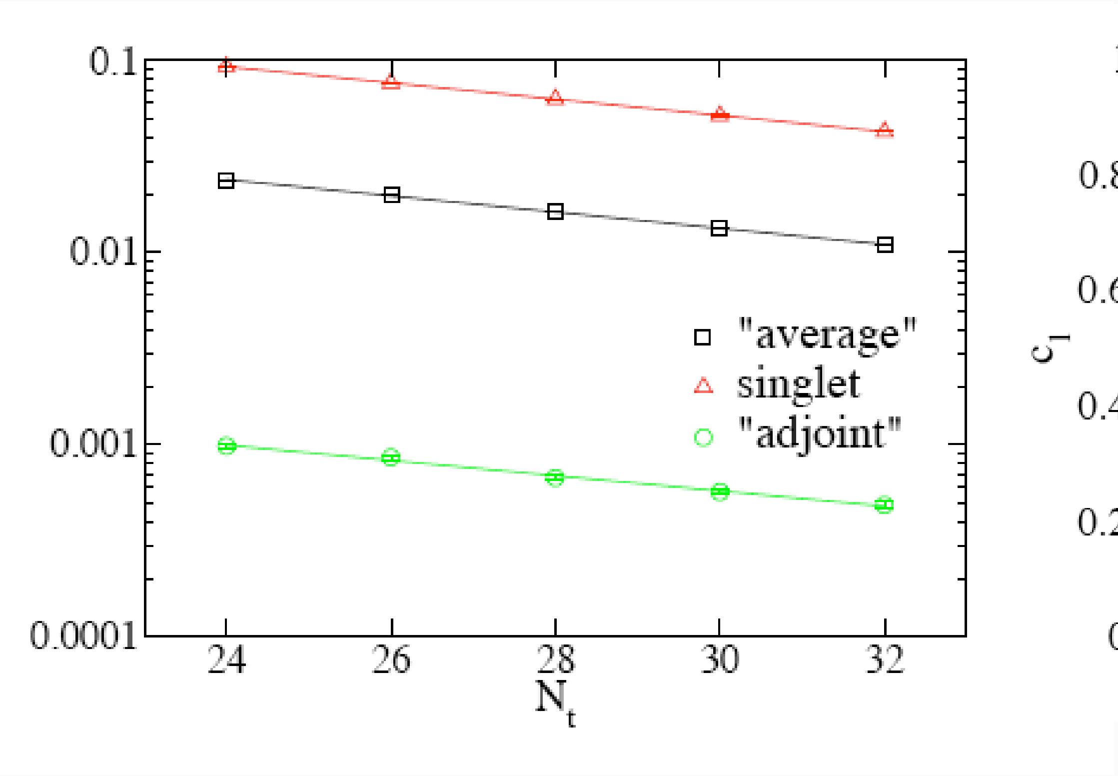
Jahn, O.P., PRD 05

$$
\begin{aligned}
e^{-F_{\bar{q}q}/T} &= \frac{1}{ZN^2}\sum_n \langle n_{\alpha\beta}|n_{\beta\alpha}\rangle \, e^{-E_n/T} \\
e^{-F_1/T} &= \frac{1}{ZN^2}\sum_n \langle n_{\delta\gamma}|\hat{U}_{\gamma\delta}(\mathbf{x},\mathbf{y})\hat{U}^{\dagger}_{\alpha\beta}(\mathbf{x},\mathbf{y})|n_{\beta\alpha}\rangle \, e^{-E_n/T} \\
e^{-F_1/T} &= \frac{1}{ZN^2}\sum_n \langle n_{\delta\gamma}|\hat{U}^{a}_{\gamma\delta}(\mathbf{x},\mathbf{y})\hat{U}^{\dagger a}_{\alpha\beta}(\mathbf{x},\mathbf{y})|n_{\beta\alpha}\rangle \, e^{-E_n/T}
\end{aligned}
$$

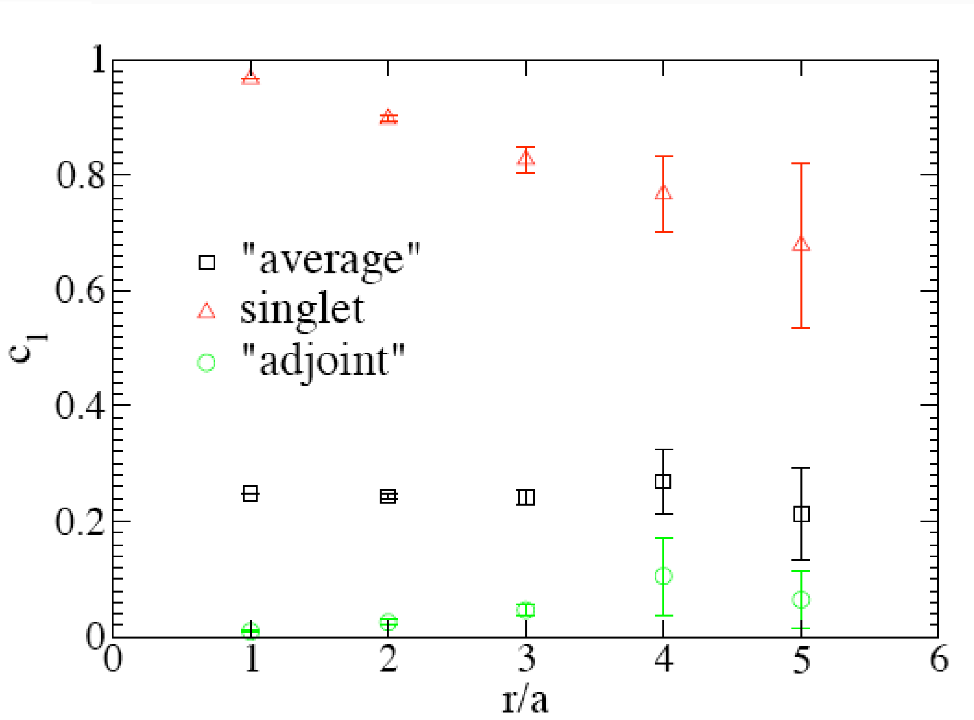
- energy eigenvalues: usual (T=0) colour singlet potential in all three channels
- non-vanishing matrix elements in singlet and octet channel
- matrix elements path/gauge dependent

Numerical demonstration in 3d SU(2), T=0 limit

allows to isolate ground state energy and matrix element for each channel



correlators, r/a=1



matrix elements

- difference between $F_1, F_{\bar{q}q}$ is exclusively in gauge fixing function!
- binding energies from F_1, U_1 artefacts?!

A real time static potential for finite T quarkonia

Laine, O.P., Romatschke, Tassler JHEP 07

- generalise effective theory approach to finite T, additional scales $\pi T, gT, g^2T$, ...
- starting point: finite T correlator related to quarkonium spectral function

$$\rho(Q) = \frac{1}{2}\left(1 - \mathrm{e}^{-\frac{q^0}{T}}\right)\tilde{C}_>(Q)$$

$$\tilde{C}_>(Q) \equiv \int dt \int d^3x\, \mathrm{e}^{iQx}\langle J^\mu(x) J_\mu(0)\rangle_T, \qquad J^\mu(x) = e_q\bar{\psi}(x)\gamma^\mu\psi(x)$$

properties at large M: zero momentum, point splitting

→ $C_>(t,\mathbf{r}) \propto W_E(it,\mathbf{r})$ Wilson loop in Minkowski-time

point splitting technical, not physical: $\tilde{C}_>(Q) \equiv \int dt\, \mathrm{e}^{iq^0x} C_>(t,\mathbf{0})$

‘exact’ evolution equation for the correlator:

$$\left\{ i\partial_t - \left[2M + V_>(t,r) - \frac{\nabla^2_{\mathbf{r}}}{M} + O\left(\frac{1}{M^2}\right) \right] \right\} C_>(t,\mathbf{r}) = 0$$

potential $\equiv$ coefficient scaling as $O(M^0)$ in t-derivative of correlator

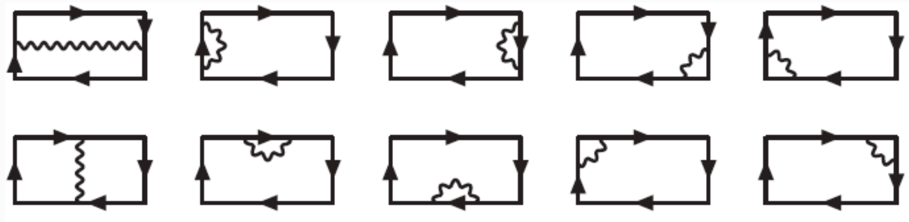
required scale hierarchy: $g^2 M < T < gM$

absorb heavy mass by rescaling, leading behaviour:

➡ $$i\partial_t W_E(it,\mathbf{r}) = V_>(t,r) W_E(it,\mathbf{r})$$

HTL resummed perturbation theory, large time limit:

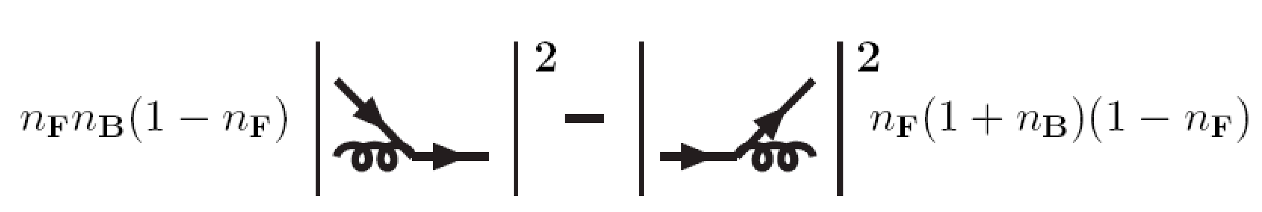
(non-relativistic: $E \ll p \quad \leftrightarrow \quad t \gg r$)



$$V_>(\infty, r) = -\frac{g^2 C_F}{4\pi}\left[m_{\rm D} + \frac{\exp(-m_{\rm D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\,\phi(m_{\rm D} r)\ ,$$

$$\text{with} \qquad \phi(x) = 2\int_0^\infty \frac{{\rm d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

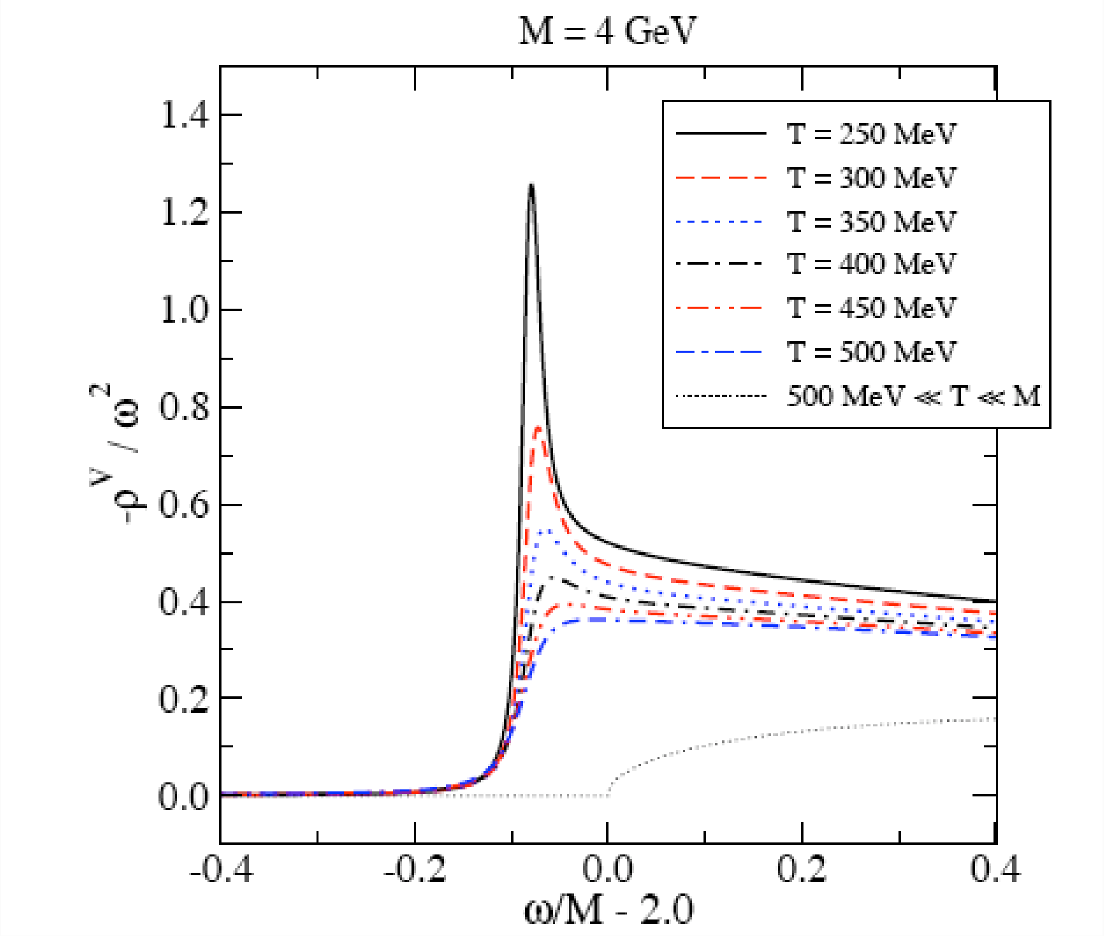
- real part familiar, Debye screening
- imaginary part due to Landau damping:
  nearly static gluons emitted/absorbed by hard particles in plasma



Result for the spectral function

Laine; Burnier, Laine, Vepsäläinen

Insert $V_>(\infty, r)$ in time-dependent Schrödinger equation, solve, Fourier transform



-qualitative features as in Matsui, Satz picture;

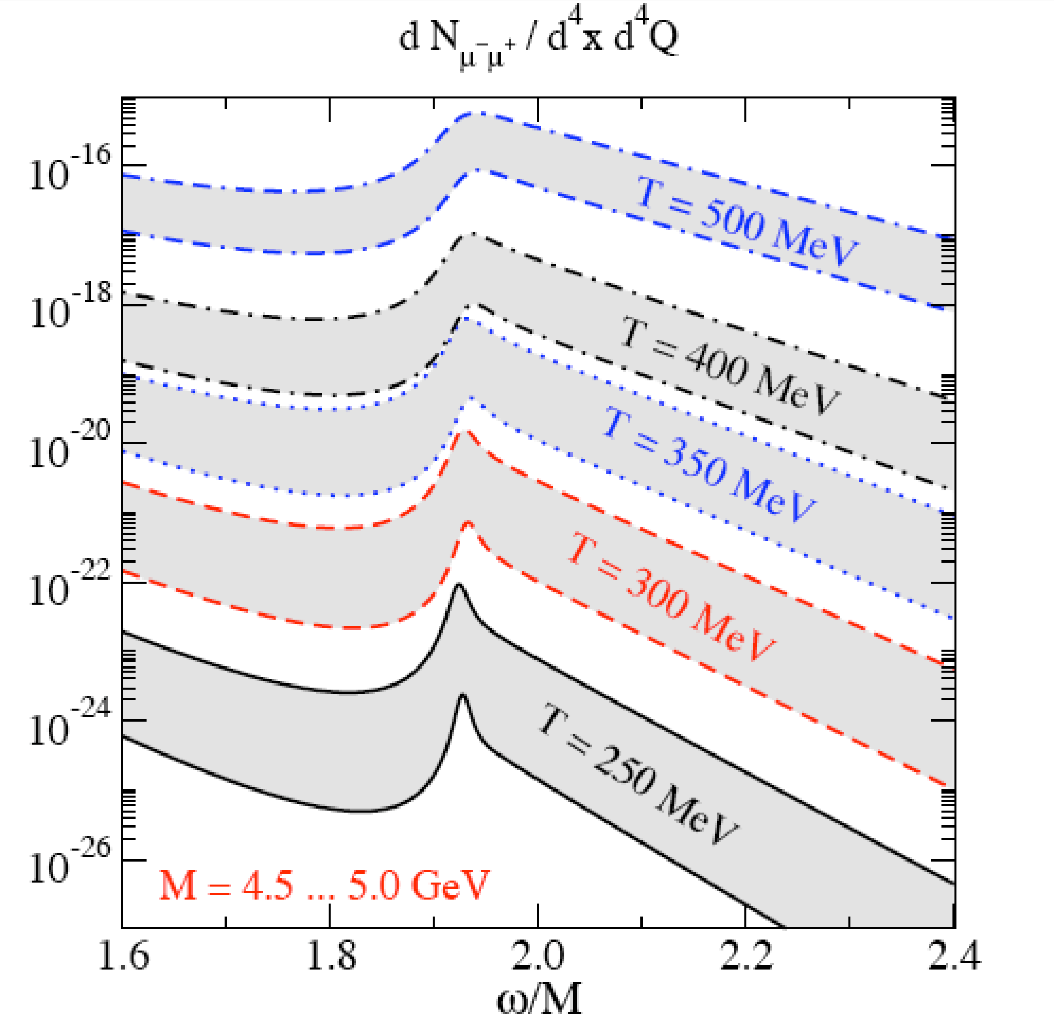
-melting temps. consistent with potential models within ~50 MeV

Dilepton production rate

$$\frac{\mathrm{d}N_{\mu^+\mu^-}}{\mathrm{d}^4x\mathrm{d}^4Q} \propto \frac{\rho_V(\omega)}{\omega^2}e^{-\frac{\omega}{T}}$$

from HTL-resummed perturbation theory

Burnier, Laine, Vepsäläinen



Non-perturbative effects?

- Wilson loop in Minkowski time not calculable on the lattice
- scales in perturbative expansion:
  vertices g^2T, cut-off Λ, 3d confinement scale m~ g^2T
- HTL resummation accounts for effect of hard modes on soft modes,
  perturbative corrections ~ g^2T/Λ
- non-perturbative corrections from infrared modes ~ g^2T/m

→ test for non-pert. infrared corrections via classical lattice simulations !

cf. sphaleron rate in e.-w. theory, plasma instabilities...

Classical limit in perturbation theory

$$V_>(\infty, r) = -\frac{g^2 C_F}{4\pi}\left[m_{\rm D} + \frac{\exp(-m_{\rm D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\,\phi(m_{\rm D} r)$$

re-instate factors of $\hbar$ in perturbative calculation: $g^2 \to g^2\hbar, \quad \frac{1}{T} \to \frac{\hbar}{T}$

$$\lim_{\hbar \to 0} V_>(\infty, r) = -\frac{i g^2 T C_F}{4\pi}\phi(m_D r)$$

- only the imaginary part survives in the classical limit !
- test for non-perturbative effects in this sector

Imaginary part from classical lattice simulations

Laine, O.P., Tassler, JHEP 07

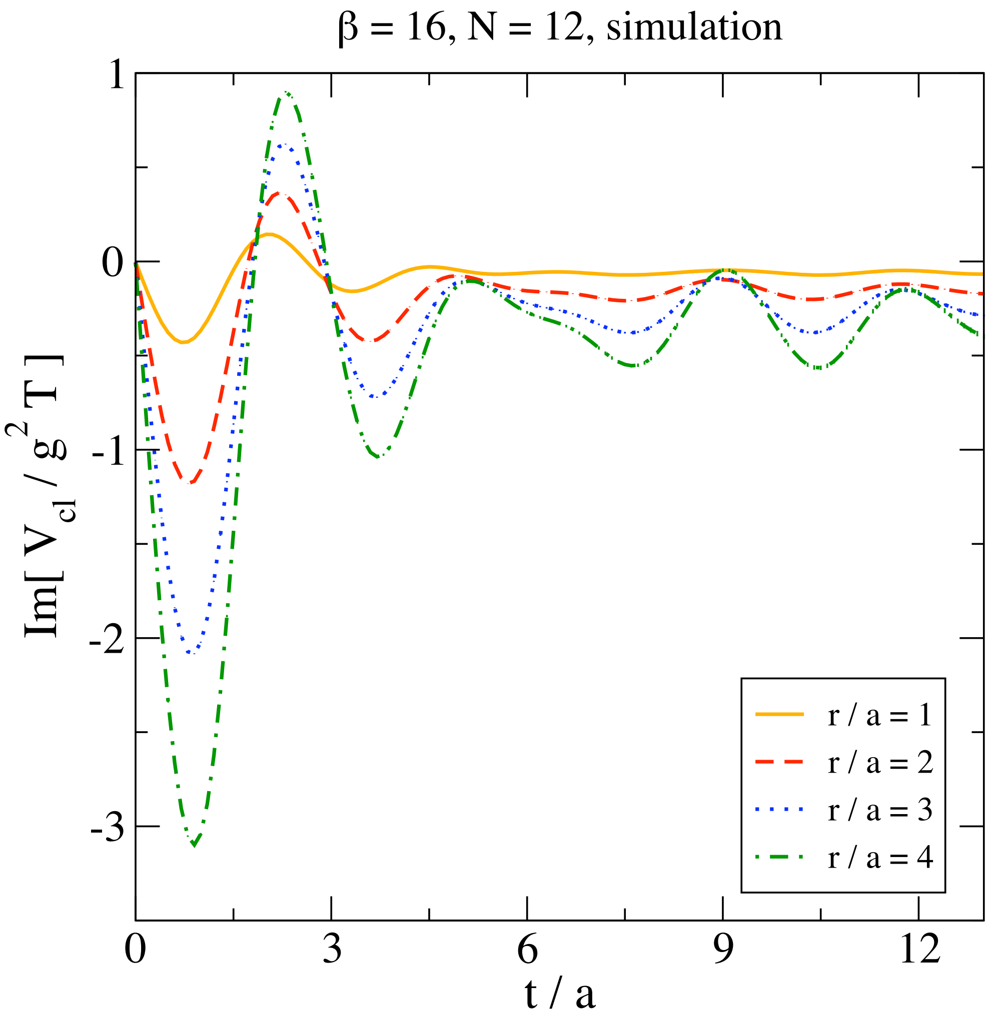
- Hamiltonian approach: t continuous, discretise on 3d lattice
- temporal gauge; conjugate fields ➡ $U_i(\mathbf{x},t), \quad \dot{U}_i(\mathbf{x},t) = iE_i(\mathbf{x},t)U_i(\mathbf{x},t)$
- Gauss constraint on non-abelian charges
- quantum treatment of UV modes via HTL effective theory possible

$$Z = \int \mathcal{D}U_i\,\mathcal{D}E_i\,\delta(G)e^{-\beta H}, \quad \beta = \frac{2N}{g^2Ta}, \quad H = \frac{1}{N}\sum_x\left[\sum_{i<j}\mathrm{Re\,Tr}\,(1-U_{ij})+\frac{1}{2}\mathrm{Tr}\,(E_i^2)\right]$$

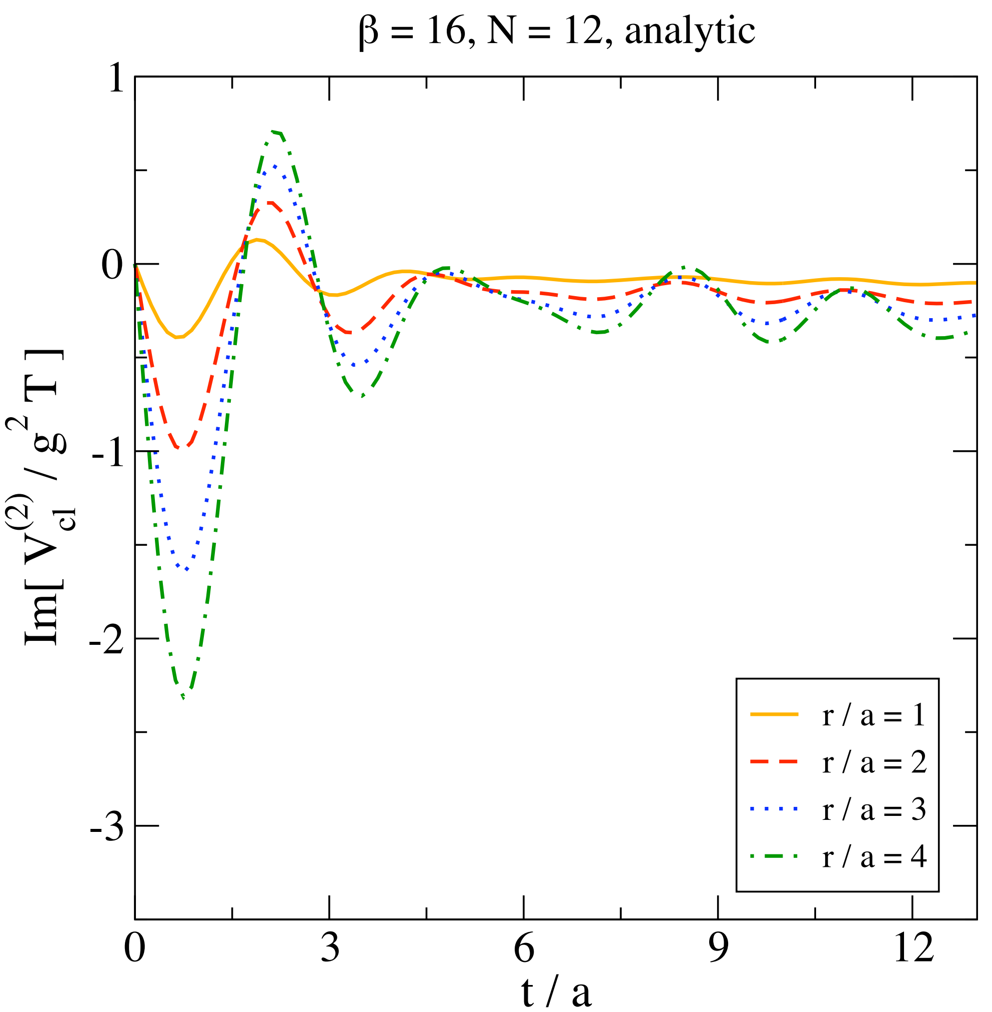
real time evolution: $\dot{U}_i(x) = iE_i(x)U_i(x)\ , \quad \dot{E}_i^a(x) = -2\,\mathrm{Im\,Tr}\,[T^a\sum_{|j|\neq i}U_{ij}(x)]$

➡ $$C_{\mathrm{cl}}(t,r) = \frac{1}{N}\mathrm{Tr}\,\langle U_r^\dagger(t)U_r(0)\rangle$$

$$V_{\mathrm{cl}}(t,r) = \frac{i\partial_t C_{\mathrm{cl}}(t,r)}{C_{\mathrm{cl}}(t,r)}$$



purely classical simulation



classical lattice perturbation theory

non-perturbative strengthening of damping

Conclusions

- many potential models for finite T quarkonia little connection to QFT
- reformulation in terms of effective QFT

  ➡ new real-time static potential

- solution possible in HTL-resummed perturbation theory

  ➡ Debye screening and Landau damping (Im V)

- Im V calculable non-perturbatively in classical lattice simulations; comparison: HTL pert. theory captures qualitative physics
- Still needed: non-pert. check for Re V
- cf. finite T pNRQCD, Brambilla et al.; Escobedo, Soto